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Thermal spreading resistance of hollow hemi-sphere with internal convective cooling

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ABSTRACT

The 2-D steady-state heat transfer in a hollow hemi-sphere subjected to an arbitrary general heat flux on its pole, and convective cooling at the inner surface, is studied analytically. Closed-form mathematical expressions for temperature distribution and non-dimensional thermal resistance as a function of radii ratio, contact angle, and the Biot number, are derived and presented for two cases that simulate the flux distribution from isoflux and isothermal heat sources. The analytical solution is verified by using a finite-element numerical solution, developed in a commercially-available software, and comparing the results. Moreover, it is demonstrated that the present fundamental analysis provides a general solution for other problems found in the literature, including spreading resistance in hollow spheres with insulated walls, as well as the heat source on a half-space and infinite disk with an isothermal end.

1. Introduction

In many applications, the estimation of thermal resistance plays a vital role in ensuring the proper design and operation of engineered devices. Mathematical expressions for thermal resistance are available in the literature for basic shapes in cartesian, cylindrical, and spherical coordinates. However, the problem of determining the spreading thermal resistance that occurs whenever heat leaves the heat source to a larger region cannot be evaluated by simple 1-D analytical models. This problem is encountered in different design areas, including heat sinks, cooling applications [1–7], and granular packed beds [8]. Exact solutions from analytical models are more favorable than approximate solutions from computational methods, as they offer compact expressions that can be easily and quickly evaluated. For this reason, there is notable interest in developing analytical closed-form models to estimate thermal spreading resistance. Kennedy [9] investigated the heat conduction in semiconductor devices by considering the heat transfer within a homogeneous, finite, solid cylinder from a circular source fixed at one of its ends. Different combinations of boundary conditions at the other surfaces in the domain were considered, and equations that describe the spreading resistance and temperature distribution were presented graphically. Muzychka et al. [10] used the separation of variables method to provide a general solution for thermal spreading resistance of an eccentric heat source on a rectangular flux channel. It was shown that the presented solution can be used for single and multiple discrete heat sources on isotropic and compound flux channels by using superposition. Yovanovich et al. [11] presented a closed-form solution for the non-dimensional thermal resistance of a rectangular isoflux heat source on a compound two-layer body with convective or conductive cooling at one boundary. Using the Fourier expansion method, Feng and Xu [12] developed a three-dimensional analytical model to determine the resistance of a rectangular isoflux source fixed on the top of cubic heat spreaders used in electronic cooling. Yovanovich [13] developed a general model for spreading/constriction resistance for a circular source on a finite circular cylinder with side and end cooling. The validity of the model for special cases was discussed. Yovanovich et al. [14] presented an analytical solution to estimate the thermal resistance of hollow spheres subjected to heat flux on a finite area at their poles, with the spheres assumed to be insulated from the inner surface. The authors [14] compared their results with a half-space and two-zone models. Using the Maxwell coordinate system, Rahmani et al. [15] developed a model for spreading resistance in a curved-edge heat spreader. Huang et al. [16] presented a general solution for spreading resistance of multiple heat sources on a rectangular flux channel under non-uniform convective cooling. Using the separation of variables method, Hsieh et al. [17] presented a 3-D analytical solution for the spreading resistance of centrally-positioned heat sources of a vapor chamber heat sink. Analytical solutions for spreading resistance in compound and orthotropic systems,

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Nomenclature

Subscripts

sink	related to heat sink
source	related to heat source

Greek symbols

α	half-contact angle [rad]
χ	relative disk thickness [-]
ε	radii ratio [-]
ϑ	non-dimensional contact angle [-]

Roman symbols

a	inner radius [m]
A_c	contact area [m^2]
b	outer radius [m]
Bi	Biot number [-]
c	chord, circular source radius [m]
h	convective heat transfer coefficient [W/m^2-K]
k	thermal conductivity [W/m-K]
Q	total heat transfer rate [W]
q	heat flux [W/m^2]
R	thermal resistance [K/W]
r	radius [m]
R^*	non-dimensional thermal resistance [-]
T	temperature [K]
t	thickness [m]

with and without cooling, have been reviewed by Muzychka et al. [18] for cylindrical and rectangular geometries. Yovanovich et al. [19] also reviewed analytical models to calculate thermal spreading resistance for compound disks, heat flux tubes and infinite layers in perfect contact with half-space. Razavi [20] presented a series of analytical solutions of the thermal spreading resistance for circular flux tubes and rectangular flux channels. Different boundary conditions were considered along the walls, source plane, and sink plane. The author [20] investigated the effect of the size of the heat source, thickness of the channels, and Biot number on the thermal resistance. Recently, Delouei et al. [21] investigated the steady-state heat conduction in thick hollow spheres by presenting an analytical solution that covers two boundary conditions, namely, variable heat flux and variable temperature.

One geometry that has not been investigated in the literature, is the thermal spreading resistance of a hollow hemi-sphere subjected to a heat flux on its pole and convective cooling at the inner surface. This problem becomes important, for example, in petroleum and cryogenic industries when heat leaks through the storage tanks to the stored fluids. These tanks often have a spherical shape to provide even distribution of stresses on the surfaces, and they are well-insulated. Heat may leak through spots with imperfect insulation, and that results in elevated temperatures and pressure, and in extreme cases, possible explosions. As the stored fluid inside the tank absorbs heat, natural convection inside the tank will occur. The aim of this study is to develop an analytical closed-form solution to determine the thermal spreading resistance for arbitrary isoflux and isothermal heat sources in a hollow hemi-sphere with convective heat transfer from the inner surface, and the associated temperature distribution.

2. Mathematical modeling

Consider an insulated hollow hemi-sphere subjected to arbitrary heat flux from a heat source on a finite area at the outer radius with 2α contact angle, and convective cooling at the inner surface, as shown in Fig. 1. The 2-D steady-state governing energy equation in the spherical coordinates is:

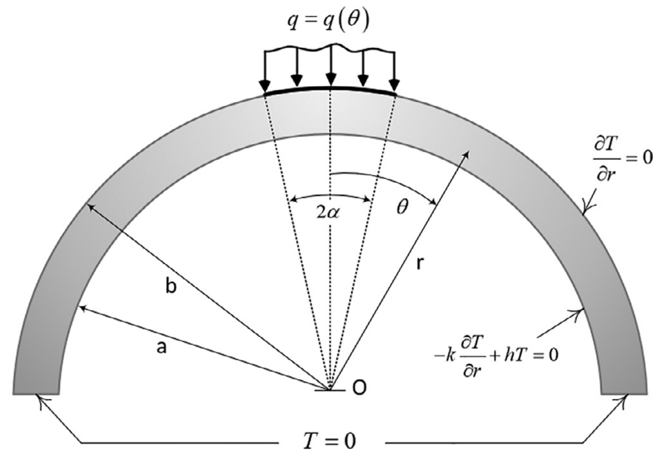

Fig. 1. Model for an insulated hollow hemi-sphere subjected to an arbitrary general heat flux on its pole, and convective cooling at the inner surface.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial T}{\partial \theta}\right) = 0 \tag{1}$$

where, r and θ are spherical coordinates. Introducing new independent variable $\mu = \cos(\theta)$, the above equation can be written as:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \mu}\left[\left(1-\mu^2\right)\frac{\partial T}{\partial \mu}\right] = 0 \tag{2}$$

Assuming isothermal conditions far away from the source contact area, the boundary conditions for the equivalent analytical model depicted by Fig. 1 are:

$$\mu = 0, a \leqslant r \leqslant b, T = 0 \tag{3a}$$

$$r = a, 0 \leqslant \mu \leqslant 1, -k\frac{\partial T}{\partial r} + hT = 0 \tag{3b}$$

$$r = b, \frac{\partial T}{\partial r} = \begin{cases} 0 \leqslant \mu \leqslant \cos(\alpha), & \frac{q(\mu)}{k} \\ \cos(\alpha) \leqslant \mu \leqslant 1, & 0 \end{cases} \tag{3c}$$

Using the separation of variables, the general form of temperature distribution is assumed to be as follows:

$$T(r,\mu) = J(r)\cdot M(\mu) \tag{4}$$

where:

$$J(r) = \sum_{n}^{\infty}\left[C_{1n}r^n + C_{2n}r^{-(n+1)}\right] \tag{5}$$

$$M(\mu) = D_{1n}P_n(\mu) + D_{2n}Q_n(\mu). \tag{6}$$

The Legendre polynomials of the second kind $Q_n(\mu)$ become infinite at $\mu = \pm 1$, therefore, they are excluded from the solution on the physical grounds. Because of the boundary condition of the first kind at $\mu = 0$ (Eq. (3a)), Legendre polynomials of even degree must be excluded, and only polynomials with odd degree should be considered ($n = 1, 3, 5, \ldots$) [22]. Accordingly, the solution can be given as:

$$T(r,\mu) = \sum_{n,odd}^{\infty}\left[A_n r^n + B_n r^{-(n+1)}\right]P_n(\mu). \tag{7}$$

Applying the second boundary condition (Eq. (3b)) in Eq. (7) gives:

$$B_n = \frac{na^{(n-1)} - \frac{h}{k}a^n}{(n+1)a^{-(n+2)} + \frac{h}{k}a^{-(n+1)}}A_n \tag{8}$$

By defining the radii ratio $\varepsilon = a/b$, the coefficient $\phi_n = B_n/A_n$, and introducing the non-dimensional Biot number $Bi = ha/k$, Eq. (7) becomes:

$$T(r,\mu) = \sum_{n,odd}^{\infty} A_n\left[r^n + \phi_n r^{-(n+1)}\right]P_n(\mu) \tag{9}$$

where:

$$\phi_n = b^{2n+1}\varepsilon^{2n+1}\Upsilon_n \tag{10}$$

$$\Upsilon_n = \frac{n - Bi}{(n+1) + Bi} \tag{11}$$

Applying the third boundary condition by substituting Eq. (3c) in Eq. (9):

$$\frac{q(\mu)}{k} = \sum_{n,odd}^{\infty} A_n\left[nb^{n-1} - (n+1)\phi_n b^{-(n+2)}\right]P_n(\mu) \tag{12}$$

Utilizing the orthogonality property of Legendre function, the following equation can be obtained:

$$\int_0^1 \frac{q(\mu)}{k}P_n(\mu)d\mu = \sum_{n,odd}^{\infty} A_n\left[nb^{n-1} - (n+1)\phi_n b^{-(n+2)}\right]\int_0^1 P_n^2(\mu)d\mu \tag{13}$$

Recalling that $q(\mu) = 0$ in $0 \leqslant \mu \leqslant \cos(\alpha)$ range, and that $\int_0^1 P_n^2(\mu)d\mu = \frac{1}{2n+1}$ [22], the coefficient A_n is determined as:

$$A_n = \frac{b^{(1-n)}}{k}\frac{(2n+1)}{\left[n-(n+1)\varepsilon^{2n+1}\Upsilon_n\right]}\int_{\cos(\alpha)}^1 q(\mu)P_n(\mu)d\mu. \tag{14}$$

After substituting Eq. (10) and Eq. (14) in Eq. (9), the expression for the temperature distribution can be written in the following form:

$$T(r,\mu) = \frac{b}{k}\sum_{n,odd}^{\infty}\frac{2n+1}{\left[n-(n+1)\varepsilon^{2n+1}\Upsilon_n\right]} \times \int_{\cos(\alpha)}^1 q(\mu)P_n(\mu)d\mu \times\left[\left(\frac{r}{b}\right)^n + \varepsilon^{2n+1}\Upsilon_n\left(\frac{r}{b}\right)^{-(n+1)}\right]P_n(\mu) \tag{15}$$

The thermal resistance can be defined as:

$$R = \frac{\overline{T}_{source} - \overline{T}_{sink}}{Q} \tag{16}$$

where, $\overline{T}_{source}$ and $\overline{T}_{sink}$ are the source and sink average temperatures, respectively. Here we take $\overline{T}_{sink}$ as the bulk temperature of the fluid inside the hemi-sphere. For convenience, this temperature is taken as $\overline{T}_{sink} = 0$. The source average temperature can be determined by:

$$\overline{T}_{source} = \frac{1}{A_c}\int\int_{A_c} T(b,\mu)dA_c. \tag{17}$$

The elemental contact area for the heat flux:

$$dA_c = 2\pi b^2\sin(\theta)d\theta = 2\pi b^2 d\mu \tag{18}$$

The total contact area, therefore, is:

$$A_c = 2\pi b^2[1-\cos(\alpha)] \tag{19}$$

Eq. (17) can be written as:

$$\overline{T}_{source} = \frac{1}{[1-\cos(\alpha)]}\int_{\cos(\alpha)}^1 T(b,\mu)d\mu \tag{20}$$

The total heat flux is determined by:

$$Q = \int\int_{A_c} q(\theta)dA_c = 2\pi b^2\int_{\cos(\alpha)}^1 q(\mu)d\mu. \tag{21}$$

Substituting Eq. (17), 21 in Eq. (16):

$$R = \frac{1}{2\pi bk[1-\cos(\alpha)]\int_{\cos(\alpha)}^1 q(\mu)d\mu}\sum_{n,odd}^{\infty}\frac{(2n+1)\left(1+\varepsilon^{2n+1}\Upsilon_n\right)}{\left[n-(n+1)\varepsilon^{2n+1}\Upsilon_n\right]} \times\int_{\cos(\alpha)}^1 q(\mu)P_n(\mu)d\mu \times \int_{\cos(\alpha)}^1 P_n(\mu)d\mu \tag{22}$$

Taking the cord, $c = b\sin(\alpha)$, as a characteristic length, a non-dimensional thermal resistance can be defined as:

$$R^* = kcR \tag{23}$$

2.1. Special cases

The present analysis can be considered as a general solution for different problems found in the literature. These special cases will be discussed in the following sections. To extend the present analysis, we consider the cases with an isothermal heat source. However, this will result in a mixed boundary condition at the outer radius (see Fig. 1 and Eq. 3c) and the problem will be difficult to solve in this form. To solve this, Yovanovich et al. [14] presented a general form for contact-area flux distribution:

$$q(\mu) = q_0[\mu - \cos(\alpha)]^{\nu} \tag{24}$$

where, q_0 is a convenient heat flux level. In the above expressions, the case when $\nu = 0$ results in a uniform heat flux, while $\nu = -1/2$ gives a flux distribution that has its minimum at the center of the contact area, the latter has the same form as a flux distribution over an isothermal circular contact situated on the surface of an isolated half-space. This flux distribution can be taken as a good approximation for the mixed boundary condition in the spherical coordinates. When considering the isothermal source condition, it is useful to define a non-dimensional contact angle as $\vartheta = 2\alpha/\pi$.

2.1.1. Thermal resistance of a hollow sphere

As mentioned in Section 1, the work done by Yovanovich et al. [14] considered the thermal resistance of a full sphere subjected to an arbitrary flux over their poles with no heat transfer from the inner surface (insulated). For the case when the convective heat transfer coefficient $h \to 0$ in Eq. (3b), the generalized analysis presented herein can be used to provide a solution for the problem investigated by Yovanovich et al. [14], as illustrated by Fig. 2a. The non-dimensional thermal resistance of sphere is twice of that expressed by Eq. (23) ($R^* = 2kcR$).

2.1.2. Spreading resistances in half-space

When the non-dimensional contact angle approaches zero, $\vartheta \to 0$ (corresponds to $c \to 0$), and $\varepsilon \to 0$, this case can be considered to be a good approximation for the thermal spreading resistance of a circular source, with a radius c, in a half-space (Fig. 2b). The spreading resistance in a half-space is taken as that defined by Eq. (23), $R^* = kcR$ [23].

2.1.3. Spreading resistance of isothermal circular source on thin infinite disk

The present model can also be used to estimate the thermal spreading resistance of a circular source, with a radius c, on a thin infinite disk that has t thickness (see Fig. 2c). This can be realized when $\vartheta \to 0$ and $h \to \infty$, and setting $\nu = -1/2$ in Eq. (24) to simulate the isothermal source condition. The definition of the resistance for this special case is $R^* = 4kcR$, as suggested by Yovanovich [13]. This resistance is a function of the relative disk thickness $\chi = t/c$. For the hemi-spherical geometry illustrated by Fig. 1, it can be defined as $\chi = (b-a)/c = (1-\varepsilon)/\sin(\alpha)$.

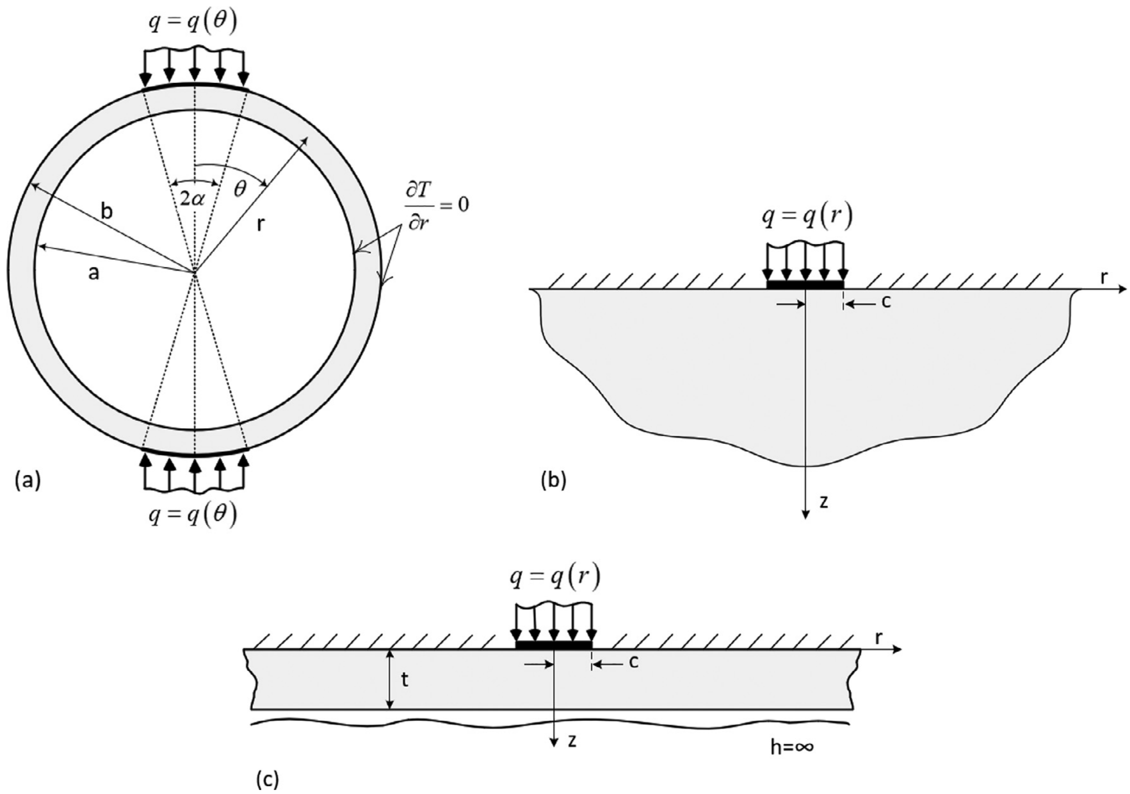

Fig. 2. Special cases: (a) Hollow sphere subjected to heat flux, (b) Spreading resistance in half-space (circular source), (c) Isothermal circular source on thin infinite disk.

3. Results and discussion

3.1. Hollow hemi-sphere

To evaluate the expressions for the temperature distribution and dimensionless thermal resistance, a computer code was written, and the solution was obtained using MATLAB software. As there is no available experimental data in the open literature to verify the analytical model and the code, the problem was solved by following a strict computational (finite-element) modeling approach using the COMSOL Multiphysics software package v5.4 (finite-element numerical method) [24]. A 3-D hemi-sphere was modeled with boundary conditions, with extremely fine mesh (to avoid any uncertainties associated with mesh dependency), and a steady-state heat transfer study was conducted by applying a constant and uniform heat flux ($\nu = 0$) at the pole, and a convective heat flux at the inner surface. The temperature distribution, normalized by the maximum temperature (at $\theta = 0$), from the analytical and numerical models is presented in Fig. 3 for different half-contact angle values ($\alpha = 1^\circ, 5^\circ, 10^\circ$, and 15°). It is clear from Figs. 3(a) to 3(d), that the results from the 2-D steady-state analytical model are in very good agreement with the 3-D numerical solution.

Fig. 4 depicts the variation of the non-dimensional thermal resistance with respect to ε and Bi, for different α values. Results shown by Figs. 4(a) to 4(d) indicate that, for low values of Bi (≤ 1), the thermal resistance increases rapidly with the increase in ε. This is because the inner wall of the hemi-sphere will act as an adiabatic surface $\frac{\partial T}{\partial r}\Big|_{r=a} \to 0$, and heat has to be spread through a restricted path in the angular direction of a thin wall (large values of ε). On the other hand, when Bi values are high enough (≥ 10), thinner walls facilitate the spreading of the heat to the inner surface, resulting in lower thermal resistances. It can also be observed that, for very small values of ε (≤ 0.2), i.e., thin spherical shells, the thermal resistance curves converge to a single value of 0.27–0.30 when α is within 1–15° range. It is worth noting that it might be misleading to conclude from Fig. 4 that the resistance is higher for large α values. In fact, although the average source temperature is lower for smaller α, hence the thermal resistance is lower, the Sine function in the definition of the non-dimensional resistance (Eq. (23)) is greater for larger α.

3.2. Hollow sphere

Table 1 compares the results from the present model and those presented by Yovanovich et al. [14] for thermal resistance of a hollow sphere with $\alpha = 5^\circ$, as discussed in Section 2.1.1. By using the expression for the heat flux distribution given by Eq. (24), it can be concluded that the results are in a very good agreement, for both isoflux ($\nu = 0$) and isothermal ($\nu = -1/2$) cases, with a maximum relative difference of $\simeq$ 2%.

3.3. Heat spreading in a half-space from a circular source

The results from the present analysis, using spherical coordinates, is compared with that presented by Bejan and Kraus [23] (cylindrical coordinates) for heat spreading in a half-space, considering a circular heat source. This comparison is presented in Table 2 for isoflux and isothermal heat sources. The relative difference between the two modeling approaches is less than 2%.

3.4. Isothermal circular source on an infinite disc

Table 3 shows the calculated non-dimensional resistance considering the approach discussed in Section 2.1.3 for isothermal circular source on an infinite disk. The results, shown for different relative thickness χ, are compared with the values presented by Yovanovich [13]. The relative difference between the results is less

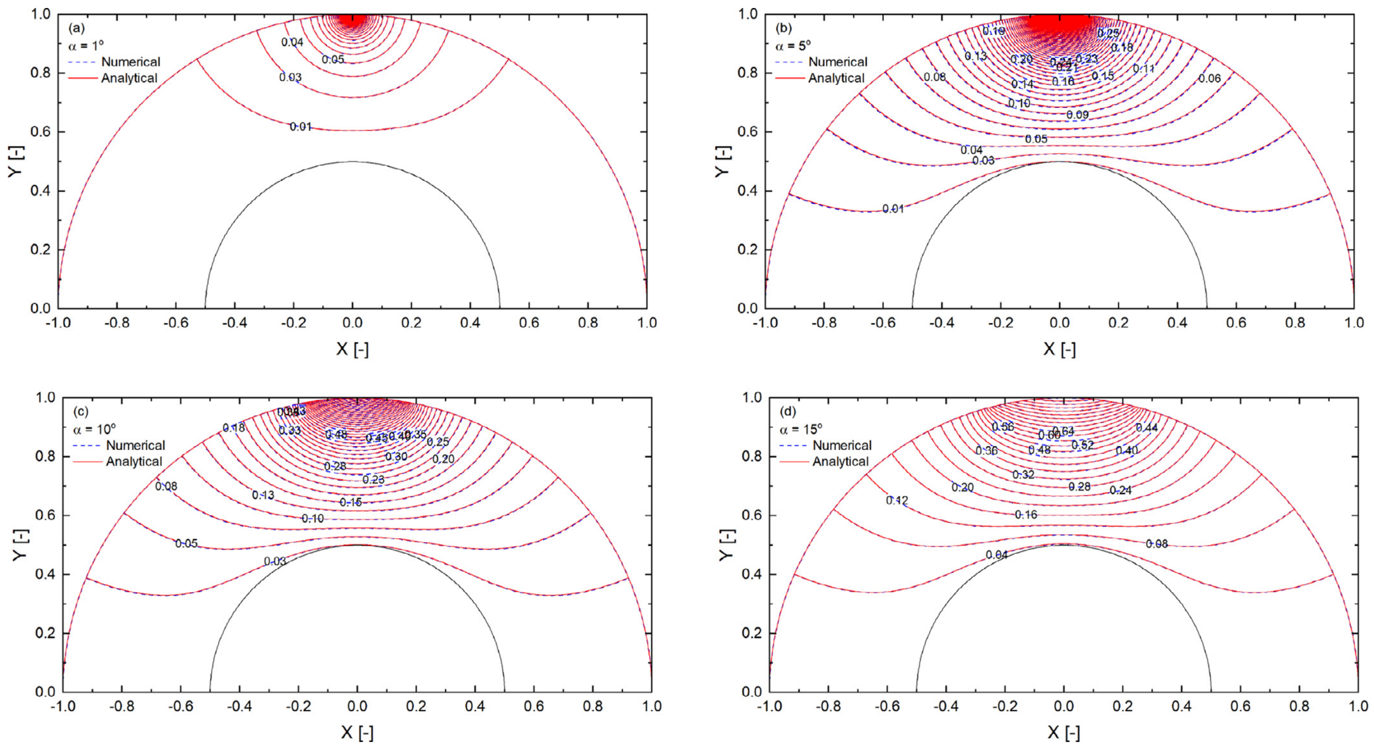

Fig. 3. Normalized temperature distribution ($T/T_{\max}$) from the 2-D analytical and 3-D numerical models for: (a) $\alpha = 1^\circ$, (b) 5°, (c) 10°, (d) 15°. For all cases, $\varepsilon = 0.5$, $Bi = 20$, $\nu = 0$.

Table 1
Non-dimensional spreading thermal resistance of a hollow sphere subjected to flux over its poles.

ε	$R^* = 2kcR$ ($\nu = 0$)			$R^* = 2kcR$ ($\nu = -1/2$)		
	Yovanovich et al. [14]	Present analysis	Difference [%]	Yovanovich et al. [14]	Present analysis	Difference [%]
0	0.5821	0.5776	−0.8	0.5404	0.5339	−1.2
0.2	0.5831	0.5785	−0.8	0.5415	0.5348	−1.2
0.4	0.5908	0.5862	−0.8	0.5492	0.5425	−1.2
0.6	0.6200	0.6152	−0.8	0.5784	0.5715	−1.2
0.8	0.7577	0.7519	−0.8	0.7157	0.7078	−1.1
0.9	1.1263	1.1175	−0.8	1.0813	1.0705	−1.0
0.92	1.3318	1.3212	−0.8	1.2842	1.2715	−1.0
0.94	1.6896	1.6754	−0.9	1.6364	1.6201	−1.0
0.96	2.4339	2.4108	−0.9	2.3662	2.3412	−1.0
0.98	4.7372	4.6752	−1.3	4.6171	4.5532	−1.4
0.99	9.4051	9.2116	−2.1	9.1720	8.9766	−2.1

Table 2
Non-dimensional spreading thermal resistance of circular isothermal source in a half-space.

Source type	Bejan and Kraus [23]	Present analysis	Difference [%]
Isoflux ($\nu = 0$)	$8/3\pi^2$	0.2726	0.9
Isothermal ($\nu = -1/2$)	1/4	0.2542	1.7

Table 3
Non-dimensional spreading thermal resistance isothermal circular source on a thin infinite disk.

χ	$R^* = 4kcR$ ($\nu = -1/2$)			χ	$R^* = 4kcR$ ($\nu = -1/2$)		
	Yovanovich [13]	Present analysis	Difference [%]		Yovanovich [13]	Present analysis	Difference [%]
0	0.0000	0.0011	0.0	2	0.7889	0.7907	0.2
0.1	0.1089	0.1014	−6.9	3	0.8559	0.8601	0.5
0.2	0.2015	0.1936	−3.9	4	0.8910	0.8970	0.7
0.3	0.2824	0.2750	−2.6	5	0.9124	0.9198	0.8
0.4	0.3532	0.3466	−1.9	6	0.9268	0.9354	0.9
0.5	0.4149	0.4090	−1.4	7	0.9372	0.9468	1.0
0.6	0.4684	0.4633	−1.1	8	0.9450	0.9555	1.1
0.7	0.5148	0.5105	−0.8	9	0.9511	0.9624	1.2
0.8	0.5551	0.5514	−0.7	10	0.9560	0.9680	1.3
0.9	0.5901	0.5871	−0.5	20	0.9779	0.9957	1.8
1.0	0.6206	0.6182	−0.4				

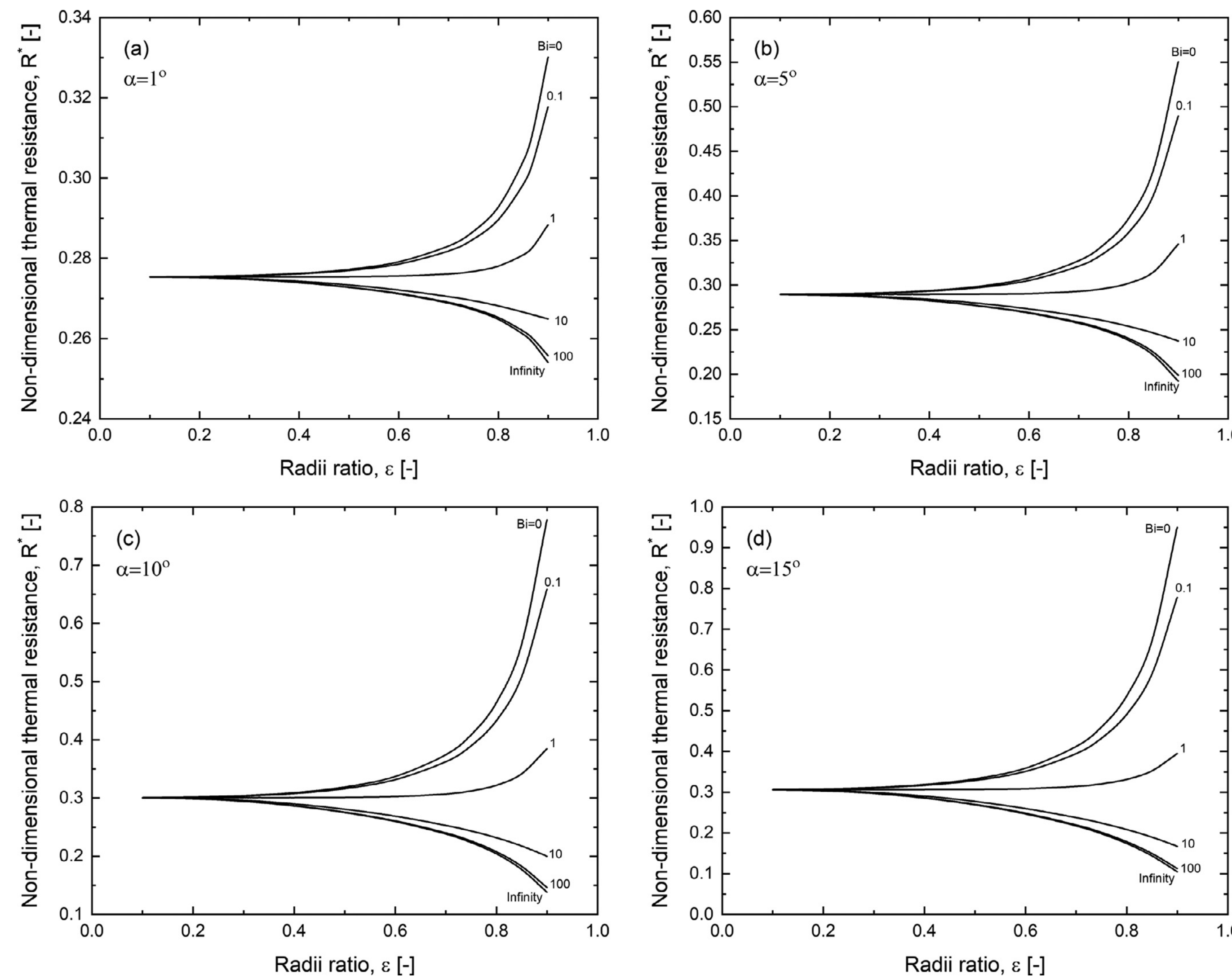

Fig. 4. Variation of non-dimensional thermal resistance with radii ratio ε, and Biot number Bi, for: (a) $\alpha = 1^\circ$, (b) $\alpha = 5^\circ$, (c) $\alpha = 10^\circ$, and (d) $\alpha = 15^\circ$.

than ±2% for $\chi = 0.4$–20 (Corresponds to $\varepsilon = 0.65$–0.99). However, when $0 < \chi \leq 0.3$, the absolute error can reach up to 7%. This discrepancy between the results within this range is because, for very small values of χ (very large values of ε), the flux distribution given by Eq. (24) cannot be considered as a good approximation for the isothermal condition. For example, for $\chi = 0.1$ ($\varepsilon > 0.99$), the relative difference between the minimum and maximum temperature within the half-contact angle α is > 100%.

4. Conclusions

The analytical expressions presented in this study provide a quick and accurate way to determine the temperature distribution and thermal spreading resistance of a hollow hemi-sphere subjected to heat flux on the its pole, with heat being dissipated from the inner surface by convection. The results show that in the case of low Biot number ($Bi \leq 1$), the thermal resistance increases rapidly for large values of radii ratio. Nevertheless, thinner walls would be favourable to dissipate the heat to the inner surface when $Bi \geq 10$. The analysis also demonstrates that smaller source contact-angle will result in higher thermal resistances.

The fundamental analysis presented herein can be used to assess the effect of heat leaks in spherical petroleum and cryogenic tanks. It has also been shown that the analytical model can be extended to include other solutions for spreading resistance in hollow spheres with insulated walls, half-space, and infinite disk with isothermal end.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Amin M. Elsafi: Conceptualization, Methodology, Investigation, Writing - original draft. **Majid Bahrami:** Conceptualization, Supervision, Funding acquisition, Writing - review & editing.

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